Proposed SUMSRI 2018 Research Projects

Graph Theory (Dr. Louis DeBiasio)

We propose a project based on certain weakenings of well-known problems in the area of (directed-, hyper-, random-, infinite-) graph-Ramsey theory, in which one seeks monochromatic substructures over all partitions (edge-colorings) of a host structure. The core problems will be chosen carefully so that these weaker versions will give insight into the general problem from which they are derived, while at the same time having the property that students will be able to make progress by studying special cases of restricted edge-colorings. While the problems will be chosen so as to be accessible to students having little to no background on advanced techniques in combinatorics, there will be many "jumping-off points" which have the potential of leading to more sophisticated results, or to the learning of advanced techniques in the subject, among them Szemerédi's regularity method, the absorbing method of Rödl-Ruciński-Szemerédi, various probabilistic tools, and linear programming.

Dynamical Systems (Dr. Alin Pogan)

We propose to conduct research in the general area of dynamical systems and pattern formation. Pattern formation is the study of mechanisms leading to the appearance of simple or complex spatial patterns. It is motivated by the existence of similar patterns in seemingly dissimilar systems such as vegetation patterns, phase separation problems, etc.

The main focus of this project is to study the existence and stability properties of stationary structures in a class of systems where a reaction-diffusion equation is coupled to a conservation law:

$$\begin{cases} u_t = \nabla \cdot \left[a(u,v)\nabla u + b(u,v)\nabla v\right], \\ v_t = \Delta v + f(u,v). \end{cases} \tag{1}$$

Here the functions a, b and f are of class $C^3(\mathbb{R}^2, \mathbb{R})$, $a(u,v) \geq a_0 > 0$, and the quantities u and v are considered to be scalar- or vector-valued. The generic quantity u is conserved under suitable decay or boundary conditions. Our general model includes systems such as the Keller-Segel model for chemotaxis. In this model u measures the concentration of a cell population, and v the concentration of a chemical produced by the bacteria. Motility of bacteria may depend on concentrations and the direction of motion on gradients of the chemical. The chemotactic behavior is typically encoded in a function $b(u,v) = -uk(u,v)$, with chemotactic sensitivity $k > 0$. The function f in (1) models concentration-dependent production and degradation of the chemical v. The simplest case, $a = 1, k = \chi$, and $f(u,v) = u - v$ is the classical Keller-Segel model.
The general model (1) also includes phase separation models arising in thermodynamics; these are conversion reaction models used to study precipitation kinetics responsible for the formation of spikes in Liesegang patterns and models that arise in biological contexts, for instance in modeling the polarization of motile eukaryotic cells in response to external signals.